CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2012 series

4037 ADDITIONAL MATHEMATICS

4037/12 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2012 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

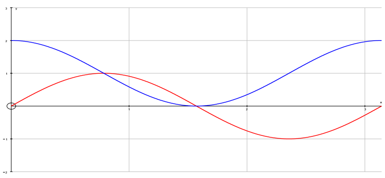
EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1 **(i)** $\left|\begin{pmatrix}24\\7\end{pmatrix}\right| = 25$	M1 A1 [2]	M1 for a complete method to find and the modulus
(ii) $4\lambda - \mu = 21$ $3\lambda + 2\mu = 2$ $\lambda = 4$ and $\mu = -5$	M1 DM1 A1 [3]	M1 for equating like vectors once DM1 for solving simultaneous equations
2 **(i)** $\frac{1}{2}\begin{pmatrix}1.5 & 1\\1 & 2\end{pmatrix}$	B1 B1 **[2]**	B1 for reciprocal of determinant B1 for matrix
(ii) $A = \begin{pmatrix}2 & -1\\-1 & 1.5\end{pmatrix}^{-1}\begin{pmatrix}1 & 6\\-0.5 & 4\end{pmatrix}$ $= \frac{1}{2}\begin{pmatrix}1.5 & 1\\1 & 2\end{pmatrix}\begin{pmatrix}1 & 6\\-0.5 & 4\end{pmatrix}$	M1	M1 for correct use of inverse matrix – must be using pre-multiplication with their inverse, must see an attempt to multiply out.
$= \frac{1}{2}\begin{pmatrix}1 & 13\\0 & 14\end{pmatrix}$ or $\begin{pmatrix}0.5 & 6.5\\0 & 7\end{pmatrix}$	A2,1,0 [3]	–1 each error

3 **(i)** $= \frac{\cos (}{\sin (} + \frac{\sin (}{1 + \cos (}$	B1	B1 for $\cot\theta = \frac{\cos\theta}{\sin\theta}$
$\cos (+ \cos^2 (+ \frac{〖\sin〗↑2 ( )}{\sin((1 + \cos()}$)	M1	M1 for attempt to add fractions
= (("cos" "(" + 1"))/("sin" "(" ("cos "("	M1	M1 for use of identity
$= \frac{1}{\sin (} = \text{cosec}($	M1 A1 [5]	M1 for algebra/simplification Must see cosec θ for A1
Alternative scheme:		
$= \frac{1}{\tan (} + \frac{\sin (}{1 + \cos (}$		
= (("1" "+" "cos" 〖"(") +" "tan" "(" " " "sin" "(		
= ("1" "+" "cos" "(" + " (〖"sin" 〗↑"2" "(")/"c	M1	M1 for attempting to add fractions
= ("cos" "(" "+" 〖"cos" 〗↑"2" "(" +" 〖"sin" 〗		
= (("cos" "(" + 1"))/("sin" "(" ("cos" "(" " "+ 1"	B1	B1 for $\tan\theta = \frac{\sin\theta}{\cos\theta}$
$= \frac{1}{\sin (} = \text{cosec } \theta$		
	M1	M1 for use of identity
	M1 A1	M1 for algebra/simplification Must see cosec θ for A1
(ii) Gives cosec $\theta = 0.5$, leads to sin $\theta = 2$ which has no solutions.	B1 [1]	Needs an explanation

4 **(i)** $\log_a p + \log_a q = 9$ $2\log_a p + \log_a q = 15$ $\log_a p = 6$ and $\log_a q = 3$	B1 B1 M1 A1 [4]	 M1 for solution of the two equations A1 for both
Or $a^9 = pq$ $a^{15} = p^2q$ $a^6 = p$ which leads to $\log_a p = 6$	B1 B1 M1	 M1 for complete solution of the two equations
$a^3 = q$ which leads to $\log_a q = 3$	A1	A1 for obtaining both in correct log form
Or $\log_a p^2q - \log_a pq = 6$	M1	M1 for $\log_a p^2q - \log_a pq = 6$
$\log_a \frac{p^2q}{pq} = 6$, $\log_a p = 6$	B1	B1 for $\log_a \frac{p^2q}{pq} = 6$
$\log_a pq = \log_a p + \log_a q = 9$ so $\log_a q = 3$	B1 A1	B1 for $\log_a pq = \log_a p + \log_a q = 9$ A1 for both
(ii) $\log_p a + \log_q a = \frac{1}{\log_a p} + \frac{1}{\log_a q}$, $= 0.5$	M1, A1 [2]	M1 for change of both to base a logarithm
5 Using $x = 6 + 2y$ or $y = \frac{x-6}{2}$	M1	M1 for attempt to obtain an equation in one variable.
$y^2 + 4y - 12 = 0$ or $x^2 - 4x - 60 = 0$	M1	M1 for reducing to a three term quadratic equated to zero
$(y + 6)(y - 2) = 0$ or $(x + 6)(x - 10) = 0$	DM1	DM1 for correct attempt to solve, must be from points of intersection
leading to $y = -6$, $y = 2$ and $x = -6$, $x = 10$	A1 A1	A1 for each correct pair
$AB = \sqrt{16^2 + 8^2}$ $= \sqrt{320}$, $8\sqrt{5}$ or 17.9	M1 A1 [7]	M1 for correct attempt to use Pythag. A1 Allow in any of these forms

6		If $\sin 15°$ is not used, then no marks available
	B1	B1 for correct statement of the sine rule
or equivalent	M1	M1 for correct manipulation to obtain sin u = an expression in surd form
	M1	M1for attempt to obtain $2\sqrt{2}$, $\sqrt{18}\,\mathrm{l}\sqrt{2}$ or reasonable attempt at simplification of their numerator
$\theta = \dfrac{2\sqrt{2}}{3\sqrt{2}+4}$	M1	M1 for attempt to rationalise, must see an attempt at simplification.
$\sin (= 6 - 4\sqrt{2}$	A1 [5]	
7 **(i)** BC, BE, EC: $y - 4 = m(x - 8)$ or $y - 8 = m(x - 6)$	M1	M1 for attempt to obtain the equation of BC, BE, EC, (gives $y = 20 - 2x$)
AD, AE: $y - 4 = -\dfrac{1}{m}(x + 5)$	M1	M1 for attempt to obtain the equation of AD, AE, (gives $2y = x + 13$)
For D, $y = 8$ and $x = 3$	B1, A1	B1 for $y = 8$, allow anywhere A1 for $x = 3$
For E, $40 - 4x = x + 13$ or equivalent leading to $x = 5.4$, $y = 9.2$	M1	M1 for attempt at the point of intersection of BE with AD, not dependent.
	A1 [6]	A1 for both
(ii) Area = $\dfrac{1}{2}(13 + 3) \times 4$ or = $\dfrac{1}{2}\begin{vmatrix} 3 & 6 & 8 & -5 & 3 \\ 6 & 8 & 4 & 4 & 8 \end{vmatrix}$	M1	M1 for a correct attempt at the area – allow odd arithmetic slip
= 32	A1 [2]	

8 **(i)** Area = $\frac{1}{2}\,18^2 \sin 1.5 - \frac{1}{2}\,10^2(1.5)$	M1	M1 for attempt at area of a sector with $r = 10$
$= 161.594 - 75$	M1	M1 for attempt at area of triangle with correct lengths used
$= 86.6$	A1 [3]	
(or area of triangle = $\frac{1}{2} \times 24.539 \times 13.170$)		
(ii) $AC = 15$ or 10×1.5 $LBD = 36 \sin 0.75$	B1 M1	B1 for AC M1 for correct attempt at BD – can be given if seen in (**i**)
$BD = \sqrt{18^2 + 18^2 - (2 \times 18 \times 18 \cos 1.5)}$ $= 24.5$		
Perimeter = $15 + 24.5 + 16$ $= 55.5$	M1 A1 [4]	M1 for attempt to obtain perimeter
9 **(a)** **(i)** 	B1 B1 B1 B1 [4]	B1 for either correct amplitude or period for $y = \sin 2x$ B1 for $y = \sin 2x$ all correct B1 for translation of +1 parallel to y-axis or correct period for $y = 1 + \cos 2x$ B1 for $y = 1 + \cos 2x$ all correct
(ii) $x = \frac{\pi}{4}, \frac{\pi}{2}$	B1, B1 [2]	Allow in degrees
(b) **(i)** Amplitude = 5, Period = $\frac{\pi}{2}$ or 90°	B1,B1 [2]	B1 for each
(ii) Period = $\frac{\pi}{3}$ or 60°	B1 [1]	

10 **(i)** $\text{f}\left(\frac{1}{2}\right): \frac{3}{2} + \frac{a}{2} + b = 0$	M1	M1 for use of $x = \frac{1}{2}$ and equating to
$\text{f}'(x) = 12x^2 + 8x + a$	M1	M1 for differentiation
$\text{f}'\left(\frac{1}{2}\right): 3 + 4 + a = 0$	M1	M1 for attempt to obtain $a = -7$ from $\text{f}'\left(\frac{1}{2}\right)$
Leading to $a = -7$ and $b = 2$	A1 A1 [5]	
(ii) $\text{f}(-3) = -49$	M1 A1 [2]	M1 for use of $x = -3$ in either the remainder theorem or algebraic long division.
(iii) $\text{f}(x) = (2x - 1)(2x^2 + 3x - 2)$	M1, A1 [2]	M1 for attempt to obtain quadratic factor
(iv) $\text{f}(x) = (2x - 1)(2x - 1)(x + 2)$ Leading to $x = 0.5, -2$	B1 B1 [2]	B1 for each – must be correct from work

11 EITHER		
(i) $= \frac{10x}{(1 + x^2)^2}$ or $\frac{dy}{dx}$ $5x^2(-2x(1+x^2)^{-2}) + (1+x^2)^{-1}\,10x$	M1 A2,1,0 A1 [4]	M1 for attempt to differentiate a quotient –1 each error
(ii) Stationary point at (0, 0)	B1	
$\frac{d^2y}{dx^2} = \frac{(1+x^2)^2\,10 - 10x(4x)(1+x^2)}{(1+x^2)^4}$	M1	M1 for a correct attempt to determine the nature of the turning point (allow change of sign method) – just finding the second derivative is not enough. Must have attempted to solve $\frac{dy}{dx} = 0$ If using second derivative, must be either a product or quotient for M1 together with some sort of conclusion.
When $x = 0$, $\frac{d^2y}{dx^2}$ is +ve, minimum	A1 [3]	
(iii)		
$\int \frac{x}{(1+x^2)^2}\,dx = \frac{1}{2}\frac{x^2}{(1+2^x)}(+c)$	B1 B1	B1 for $\frac{xx^2}{(1+x^2)}$, B1 for $\frac{1}{2}\frac{x^2}{(1+x^2)}$
$\int_{-1}^{2} \frac{x}{(1+x^2)^2}\,dx = \frac{1}{2}\left[\frac{4}{5} - \frac{1}{2}\right]$	M1	M1 for correct use of limits in an attempt at integration
$= 0.15$	A1 [4]	

11 OR **(i)** $\frac{dy}{dx} = \frac{(x^2-2)2Ax-(Ax^2+B)2x}{(x^2-2)^2}$ $= \frac{2x(Ax^2-2A-Ax^2-B)}{(x^2-2)^2}$ $= \frac{2x(2A+B)}{(x^2-2)^2}$ $\frac{dy}{dx} = (x^2-2)^{-1}\,2Ax + (-2x)(x^2-2)^{-2}\,(Ax^2+B)$	M1 A2,1,0 A1 [4]	M1 for attempt to differentiate a quotient –1 each error **Answer given**
(ii) $5 = 2A + B$ $3 = A + B$ Leading to $A = 2$, $B = 1$	M1 M1 A1 [3]	M1 for use of conditions once M1 for use of conditions a second time and attempt to solve resulting equations
(iii) when $\frac{dy}{dx} = 0$, $x = 0$ $y = -\frac{1}{2}$ $\frac{d^2y}{dx^2} = \frac{(x^2-2)^2(-10)-(-10x)\,4x(x^2-2)}{(x^2-2)^4}$ When $x = 0$, $\frac{d^2y}{dx^2}$ is –ve $\therefore$ max	B1 √B1 M1 A1 [4]	B1 for correct x √B1 for $y = -\frac{B}{2}$ M1 for a correct attempt to determine the nature of the turning point (allow change of sign method) – just finding the second derivative is not enough. Must have attempted to solve $\frac{dy}{dx} = 0$ If using second derivative, must be either a product or a quotient for M1 together with some sort of conclusion. A1 for a correct conclusion from completely correct work.